Models and examples in linear optimization

Example 2. It is confirmed that the following number of staff members are needed for servicing the customers of the Sofia airport according the time of day:

Time gap	0-4	4-8	8-12	12-16	16-20	20-24
Needed staff	70	130	120	150	200	100

There are 6 work shifts for the staff, which begin start in 0, 4, 8, 12, 16 and 20 Hours and last for 8 hours each. The management needs to decide how many people to assign to each shift so that the staffing needs are met by hiring the minimal number of workers (and their costs and salaries) possible.

Solution:

Let the variable X_1 show the number of staff for the first shift (which starts from 0), the variable X_2 show the number of staff for the second shift (which starts from 4), the variable X_3 show the number of staff for the third shift (which starts from 8), the variable X_4 show the number of staff for the fourth shift (which starts from 12), the variable X_5 show the number of staff for the fifth shift (which starts from 16), the variable X_6 show the number of staff for the sixth shift (which starts from 20). Those working in the first and sixth shifts cover the needs of the 0-4 time gap, those in the first and second – for the 4-8 gap etc, with the fifth and sixth shifts handling the final 20-24 time gap.

That way we get the following linear optimization problem:

Min $X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

With constraints:

$$X_1 + X_6 \geq 70$$

$$X_1 + X_2 \geq 130$$

$$X_2 + X_3 \geq 120$$

$$X_3 + X_4 \geq 150$$

$$X_4 + X_5 \geq 200$$

$$X_5 + X_6 \geq 100$$

$$X_j \geq 0 \text{ and whole for } j=1, 2, 3, 4, 5 \text{ and } 6$$

Answer

The answer to the problem is as follows:

A total number of 390 people are hired which, which are spread among the shifts as follows:

Shift	0-8	4-12	8-16	12-20	16-24	20-04
Staff size	70	70	50	100	100	-

That way the number of hired people cover all hourly requirements precisely, with the only exception being that the time gap 4-8 has 10 more than needed.

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